4.4a Modeling and Optimization

Strategy for solving max/min problems

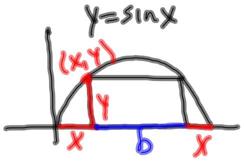
1. Understand the problem.
2. Find a function to model the problem. Use pictures, label variables, constants.
3. Graph the function. Find the domain that makes sense
4. Find the critical points and endpoints
5. Use the first or second derivative test to identify maximums and minimums.
6. Answer the original question.

Oct 18-6:48 PM

A rectangle is to be inscribed under one arch of a sine curve. What is the largest area the rectangle can have, and what dimensions give that area?

1. guess and check with 4.4 rectangle under sine curve.tns

$y = \sin x$



max area

$A = b \cdot h$

$A = (\pi - 2x)\sin x$

$A' = (\pi - 2x)\cos x + \sin x(-2)$

$0 = (\pi - 2x)\cos x - 2\sin x$

$x = .7105$

$a''(.7105) = -4.1543 < 0$

so max at $x = .7105$

$0 \le x \le \frac{\pi}{2}$

$h = y = \sin x$

$b + 2x = \pi$

$b = \pi - 2x$

max is $a(.7105)$

$= 1.122$

$b = \pi - 2(.7105)$

$h = \sin(.7105)$

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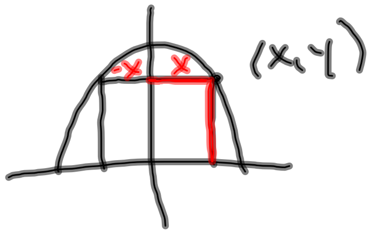
~~Solve using a deriv~~ative



$a = 2xy$

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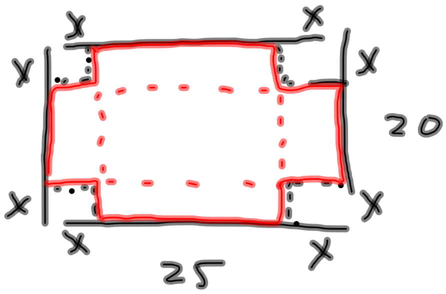
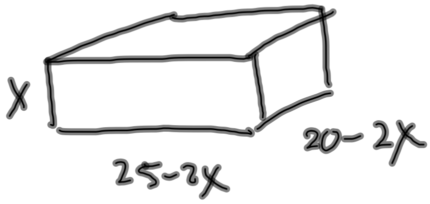
An open top box is to be made by cutting squares from the corners of a 20 by 25 inch sheet of cardboard and bending up the sides. What is the largest possible volume?

$V = l \cdot w \cdot h$





$V = x(25-2x)(20-2x)$

$V' = 12x^2 - 180x + 500 = 0$

max is

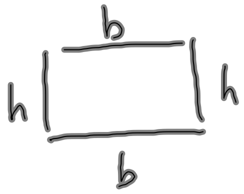
max at $x = 3.68119$

$V(3.68119) = 820.528 \text{ in}^3$

$V''(3.68119) = -91.65 < 0$

Oct 18-7:17 PM

What is the largest rectangular garden that can be enclosed with 600 feet of fence?



$2b + 2h = 600$

$b + h = 300$

$b = 300 - h$

$A = b \cdot h = (300-h)h$

$A = 300h - h^2$

$A' = 300 - 2h = 0$

max at $h = 150$

$A'' = -2 < 0$

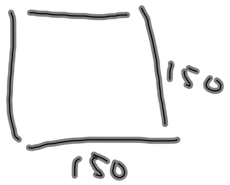
$A''(150) = -2$

max is $A(150) =$

$= 300 \cdot 150 - 150^2$

$= 22{,}500 \text{ ft}^2$

$b = 150$



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5. $y = x\sqrt{8-x^2}$

$8 - x^2 \geq 0$ $\quad 8 \geq x^2$

endpts $x = \pm\sqrt{8}$

$y' = x(-2x)\frac{1}{2}(8-x^2)^{-\frac{1}{2}} + \sqrt{8-x^2} \cdot 1$

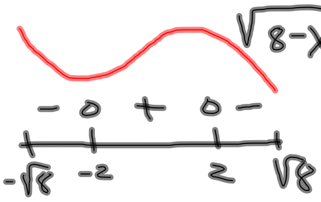
$= \dfrac{-x^2}{\sqrt{8-x^2}} + \sqrt{8-x^2} = 0$

$\sqrt{8-x^2}\sqrt{8-x^2} = \dfrac{x^2}{\cancel{\sqrt{8-x^2}}}$

$8 - x^2 = x^2$

$4 = \frac{8}{2} = 2x^2$

$x = \pm 2$



min at $x = -2$ $\quad y =$

max at $x = 2$ $\quad y =$

max at $x = -\sqrt{8}$ $\quad y =$

min at $x = \sqrt{8}$ $\quad y =$

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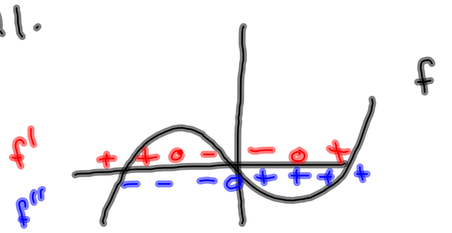
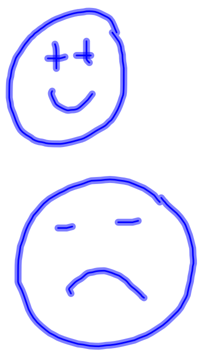
21.





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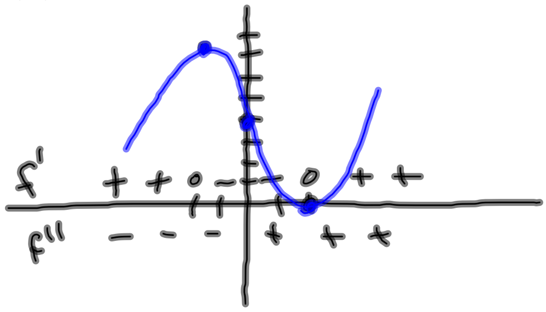
47.



33 $y = 3x - x^3 + 5$

$y' = 3 - 3x^2 = 0 \quad x = \pm 1$

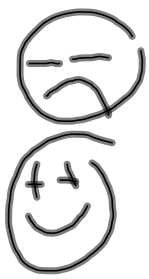
$y'' = -6x$

$y''(1) = -6$

$y''(-1) = 6$

max at $x = 1$

min at $x = -1$



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